NUMERICAL STUDY OF A MODEL OF FATIGUE WEAR AND FRACTURE

I. I. Kudish

UDC 531:539.3

A statistical model was proposed in [1] to describe fatigue wear and fracture. The model is based on a single mechanism of the growth of subsurface fatigue cracks in quasibrittle bodies under contact loading conditions. Here, it is presumed that normal and shear contact stresses are present in the contact and that residual stresses are present in the surface layers of the material. According to the model, cracks which form in the material are straight, do not interact with one another, and are randomly distributed throughout the volume of the material. Crack growth is monitored on the basis of the stress intensity factor at the tip of a given crack and is governed by a kinetic equation.

In the present investigation, we use the model in [1] to numerically study fatigue wear and fracture. We analyze the effect of the main parameters of the model on contact fatigue. The conditions required for the beginning of wear, delamination, and fracture are established and several numerical results are presented.

1. Initial Relations of the Problem. We first make use of the Paris equation describing fatigue crack growth

$$dl/dN = g_0 k_1^{2m}, \tag{1.1}$$

where k_1 is the highest load-cycle value of the stress intensity factor for normal rupture at the crack tip; ℓ is the half-length of the crack; N is the number of load cycles; g_0 and m are constants of the equation. We will assume that the statistical distribution f of the cracks with respect to their initial half-length ℓ_0 conforms to the log-normal law

$$f(0, x, y, l_0) = \begin{cases} 0, & l_0 \leqslant 0, \\ \dfrac{n(0, x, y)}{(2\pi)^{1/2}\bar{\sigma} l_0} \exp\left[-\dfrac{1}{2}\left(\dfrac{\ln(l_0) - \bar{\mu}}{\bar{\sigma}}\right)^2\right], & l_0 > 0. \end{cases} \tag{1.2}$$

Here n(0, x, y) is the volume density of cracks; $\bar{\mu}$ and $\bar{\sigma}$ are the mathematical expectation and standard deviation of the logarithm of the half-length of the cracks at the initial moment of time. We further assume that the quantities n, $\bar{\mu}$, and $\bar{\sigma}$ are independent of the coordinates (x, y) of the given material point (the x axis is directed along the surface of the body, while the y axis is perpendicular to the surface). Then the probability of the absence of fracture at the point (x, y) is as follows [1] [also see (1.12)]

$$\tilde{p}(N, x, y) = \frac{1}{2}\left[1 + \operatorname{erf}\left(\frac{\ln(\tilde{l}_{0k}) - \bar{\mu}}{\bar{\sigma}}\right)\right], \tag{1.3}$$

where erf(z) is the probability integral; $\tilde{\ell}_{0k} = \tilde{\ell}_{0k}(N, x, y)$ is the initial half-length of a crack that, after N load cycles, reaches the critical half-length $\tilde{\ell}_k = \tilde{\ell}_k(N, x, y)$ [1] [see (1.1)]:

$$\begin{gathered} \tilde{l}_{0k} = \left[\tilde{l}_k^{1/\beta} - g_0(1-m)\int_0^N k_{10}^{2m}(\tau, x, y - Y(\tau, x), \alpha)\,d\tau\right]^\beta, \quad \beta = (1-m)^{-1}, \\ \tilde{l}_k = l_k(N, x, y - Y(N, x)) = [K_{fc}/k_{10}(N, x, y - Y(N, x), \alpha)]^2, \quad k_{10} = k_1 l^{-1/2}. \end{gathered} \tag{1.4}$$

In (1.4), Y(N, x) is the linear wear of the surface of the lower contacting body (for simplicity, we will henceforth assume that only one of the contacting bodies undergoes wear); K_{fc} is cyclic fracture toughness; k_{10} is the stress-intensity factor for normal rupture at the tip of a crack of unit half-length, determined in accordance with the asymptotic monomial

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 134-141, May-June, 1991. Original article submitted June 26, 1989.

solution in [2]; α is the angle of orientation of the crack, determined from the equation [1]

$$k_2(N, x, y, \alpha) = 0 \tag{1.5}$$

(k_2 is the stress intensity factor for shear at the crack tip [2]).

By wear, we mean the conditions for which

$$I(N, y_*, l_*) > I_w, \tag{1.6}$$

while the term fracture is taken to mean the conditions for which

$$I(N, +\infty, +\infty) - I(N, y_*, l_*) \leqslant I_p \tag{1.7}$$

(ℓ_*, I_w, and I_p are empirical constants; an expression for the number of fracture products I per unit working area was presented in [1]).

According to [1], the depth of the layer destroyed by wear over N load cycles is expressed as

$$\begin{gathered} Y(N, x) = m_c^{-1} \int\limits_{\Omega_x} y\,[1 - \tilde{p}(N, x, y)]\,dy, \quad m_c = \int\limits_{\Omega_x} [1 - \tilde{p}(N, x, y)]\,dy, \\ \Omega_x = \Omega_x(N, y_*, l_*) = \{y \mid 0 < Y(N, x) - y \leqslant y_*,\ \tilde{l}_k \leqslant l_*\} \end{gathered} \tag{1.8}$$

[y_* is found from (1.6)].

The probability of the absence of fracture [1]

$$P(N) = \min_{x, y \in C\Omega_x} \tilde{p}(N, x, y). \tag{1.9}$$

Here, y_* is determined from (1.6); $C\Omega_x$ is the complement of Ω_x to the interval $(-\infty, Y(N, x))$. Fracture takes place only when condition (1.7) is satisfied. A more detailed explanation of the contact fatigue model is given in [1].

We introduced the following dimensionless variables in Eqs. (1.3)-(1.5), (1.8), and (1.9)

$$\begin{gathered} (x', y', Y') = (x, y, Y)/b, \quad (\tilde{l}'_{0k}, \tilde{l}'_k) = (\tilde{l}_{0k}, \tilde{l}_k)\, e^{-\bar{\mu}}, \\ (k'_{10}, k'_2, q^{0\prime}) = (k_{10}, k_2, q^0)/q, \ L_0 = (K_{fc}/q)^2 e^{-\bar{\mu}}, \quad G_0 = g_0 q^{2m} e^{-\bar{\mu}\beta} \end{gathered} \tag{1.10}$$

(q^0 are the residual stresses acting along the surface of the body; q and b are the maximum pressure and the half-width of the Hertzian contact). In the variables (1.10) (with the primes omitted), the relations for Y and P remain unchanged but the relations for $\tilde{\ell}_{0k}$, $\tilde{\ell}_k$, and $\tilde{P}$ take the form

$$\begin{gathered} \tilde{l}_{0k} = \left[\tilde{l}_k^{1/\beta} + (m-1)\, G_0 \int\limits_0^N k_{10}^{2m}(\tau, x, y - Y(\tau, x), \alpha)\, d\tau\right]^\beta, \\ \tilde{l}_k = L_0/k_{10}^2(N, x, y - Y(N, x), \alpha), \quad \tilde{p} = 0{,}5\left[1 + \operatorname{erf}(\bar{\sigma}^{-1} \ln(\tilde{l}_{0k}))\right]. \end{gathered} \tag{1.11}$$

Thus, the rate of wear and fracture in a unit contact depends on six dimensionless characteristics: $\bar{\sigma}$, L_0, G_0, m, q^0, and λ (the friction coefficient $\lambda = -\tau/p$, where τ and p are the shear and normal contact stresses). The initial number of dimensionless parameters will obviously be large in the case $q^0 = q^0(y)$.

2. Method of Numerical Analysis. The problem of wear and fracture can be studied numerically on the basis of the following relations [see (1.5), (1.8), (1.9), (1.11)]

$$\begin{gathered} \tilde{l}_{0k,i+1} = \{\tilde{l}_{0k,i}^{1/\beta} + \tilde{l}_{k,i+1}^{1/\beta} - \tilde{l}_{k,i}^{1/\beta} + 0{,}5\Delta N_i (m-1) G_0 [k_{10}^{2m}(N_i, x, y - \\ - Y_i, \alpha) + k_{10}^{2m}(N_{i+1}, x, y - Y_{i+1}, \alpha)]\}^\beta, \end{gathered}$$

TABLE 1

$\lvert Y(N) \rvert$, μm	Base variant	$q=1{,}96$ GPa	$q=0{,}98$ GPa	$\lambda=0{,}006$	$\lambda=0{,}0014$	$q^0=49{,}03$ MPa	$q^0=0{,}29$ GPa	$\tilde{\mu}=1$	$\tilde{\mu}=2{,}69$	$\bar{\sigma}=0{,}2$	$\bar{\sigma}=1$	$g_0=1{,}87\cdot10^{-11}$ $MPa^{-2m}\cdot m^{1-m}$	$K_{fc}=14{,}26$ $MPa\cdot m^{1/2}$
	$N\cdot10^{-8}$												
0	0,68	2,49	22,6	3,46	0,23	0,099	0,00215	0,96	0,35	1,16	0,28	1,48	0,68
0,05	2,20	7,95	72,6	14,6	0,64	0,189	0,00262	3,09	1,13	3,78	0,89	3,83	2,19
0,10	3,19	11,6	105,6	22,6	0,90	0,232	0,00280	4,50	1,65	5,53	1,30	5,58	3,19
0,15	4,11	15,0	135,8	30,1	1,13	0,267	0,00293	5,79	2,13	7,13	1,67	7,18	4,11
0,20	5,01	18,2	165,1	37,4	1,36	0,297	0,00302	7,05	2,59	8,66	2,03	8,73	5,00
	$\lvert dY/dN \rvert\cdot10^{10}$												
	5,42	1,72	0,47	0,68	20,97	143,1	39 400	3,57	12,0	2,99	12,0	3,34	5,60

$$Y_{i+1} = \frac{0{,}5}{m_{c,i+1}} \int\limits_{\Omega_{x,i}} y \left\{1 - \operatorname{erf}\left(\bar{\sigma}^{-1} \ln\left(\tilde{l}_{0k,i+1}\right)\right)\right\} dy,$$
$$m_{c,i+1} = 0{,}5 \int\limits_{\Omega_{x,i}} \left\{1 - \operatorname{erf}\left(\bar{\sigma}^{-1} \ln\left(\tilde{l}_{0k,i+1}\right)\right)\right\} dy, \tag{2.1}$$
$$P_i = P(N_i) = 0{,}5\left\{1 + \operatorname{erf}\left(\bar{\sigma}^{-1} \ln\left(\min_{x,y \in C\Omega_{x,i}} \tilde{l}_{0k,i}\right)\right)\right\}.$$

Here, the subscript i denotes the number of the step ΔN_i with respect to the number of load cycles; $N_i = N_{i-1} + \Delta N_{i-1}$; $Y_i = Y(N_i, x)$; $\tilde{l}_{0k,i} = \tilde{l}_{0k}(N_i, x, y)$; $\tilde{l}_{k,i} = \tilde{l}_k(N_i, x, y)$. This scheme makes it possible for the step ΔN_i to be chosen during the computation in relation to the behavior and values of Y_i, $\tilde{l}_{0k,i}$, and P_i.

When calculations are performed with (2.1), the angle of orientation of the crack is found from the relation [2]

$$\operatorname{tg}(2\alpha) = -2y \int\limits_a^c \frac{(t-x)\left[yp(t) + (t-x)\tau(t)\right] dt}{\left[(t-x)^2 + y^2\right]^2} \Bigg/ \left\{\frac{\pi}{2} q^0 + \int\limits_a^c \frac{\left[(t-x)^2 - y^2\right]\left[yp(t) + (t-x)\tau(t)\right] dt}{\left[(t-x)^2 + y^2\right]^2}\right\}. \tag{2.2}$$

As the angle of orientation of the crack α, we choose a solution of Eq. (2.2) that corresponds to a large value of the stress-intensity factor for normal rupture.

3. Numerical Results. We will examine the process of cyclic loading with an amplitude which is constant over time. The contact shear stresses will be determined in accordance with Coulomb's friction law with the friction coefficient λ. We henceforth assume in the calculations that the initial concentration of defects in the material (the spatial distribution of cracks in it) is uniform over its volume.

Let us analyze the results of the numerical solution of a problem on the wear of a body (in dimensional variables) made of steel ShKh15. Steel of this grade made by the standard technology is characterized by the following values [3]:

$$\bar{\mu} = 1{,}58 + \ln(\mu m), \quad \bar{\sigma} = 0{,}5,$$
$$K_{fc} = 16{,}06 \ \mathrm{MPa\cdot m^{1/2}}, \quad g_0 = 3{,}26\cdot10^{-11}\ \mathrm{MPa^{-2m}\cdot m^{1-m}}, \quad m = 1{,}59. \tag{3.1}$$

We take q = 2.94 MPa, b = 100 μm, $q^0 = 0$, and λ = 0.01. We will refer to the computational variant employing these values as the base variant. Table 1 shows data for different computational variants with five values for absolute linear wear: $|Y|$ = 0, 0.05, 0.1, 0.15, 0.2 μm. The column "computational variant" indicates the quantity determined and its value, which distinguishes the given variant from the base variant. The remaining columns show the number of load cycles N corresponding to attainment of the prescribed values by the quantity $|Y|$. The numbers N corresponding to Y = 0 coincide with incubation periods. When these numbers are exceeded, $Y \neq 0$. The last row of Table 1 shows mean values of $|dY/dN|$ (the wear

TABLE 2

$N \cdot 10^{-11}$	$\vert Y(N)\vert \cdot 10^2$, μm	$N \cdot 10^{-11}$	$\vert Y(N)\vert \cdot 10^2$, μm
0,3011	0	202,0	1,75
0,3083	0,000125	231,1	2
28,87	0,25	260,0	2,25
57,66	0,5	288,6	2,5
86,79	0,75	317,7	2,75
115,7	1	346,5	3
144,3	1,25	375,1	3,25
173,2	1,5	404,1	3,5

TABLE 3

$\vert y\vert \cdot 10^2$, μm	$k_{10} \cdot 10^{11}$, MPa	$\alpha \cdot 10^3$
0,000125	8645	1589,36
0,25	2078	1559,49
0,5	1,790	54,39
0,75	0,6938	14,67
1	0,5871	10,43
1,25	0,5575	8,438
1,5	0,5512	7,528
1,75	0,5499	6,797
2	0,5554	6,394
2,25	0,5610	6,075

TABLE 4

$N \cdot 10^{-24}$	0,384	0,393	0,400	0,407	0,414	0,421	0,435	0,442	0,449	0,456	0,479	0,486
$\vert Y\vert$, μm	0	0,558	0,594	0,609	0,616	0,624	0,747	0,768	0,776	0,783	0,935	0,943
$N \cdot 10^{-24}$	0,530	0,537	0,543	0,550	0,556	0,563	0,599	0,606	0,613	0,620	0,642	0,649
$\vert Y\vert$, μm	1,138	1,202	1,239	1,261	1,269	1,276	1,407	1,435	1,443	1,450	1,602	1,617
$N \cdot 10^{-24}$	0,656	0,663	0,685	0,692	0,699	0,706	0,714	0,728	0,735	0,742	0,771	0,778
$\vert Y\vert$, μm	1,631	1,639	1,791	1,812	1,820	1,834	1,842	1,979	1,994	2,001	2,154	2,189
$N \cdot 10^{-24}$	0,785	0,792	0,799	0,822	0,829	0,836	0,843	0,865	0,871	0,878	0,908	0,915
$\vert Y\vert$, μm	2,204	2,211	2,219	2,371	2,399	2,414	2,422	2,574	2,588	2,596	2,755	2,762
$N \cdot 10^{-24}$	0,922	0,951	0,958	0,965	0,972	0,994	1,001	1,038	1,044	1,051	1,059	1,066
$\vert Y\vert$, μm	2,770	2,929	2,965	2,980	2,987	3,132	3,147	3,299	3,334	3,349	3,357	3,364

rate $I = |dY/dL|$, where L is the length of the path of the crack, is obviously equal to $I = 0.5b^{-1}|dY/d\bar{N}|)$ averaged over the points corresponding to the indicated values of $|Y|$. Also, in Table 1 we took $\tilde{\mu} = \bar{\mu} - \ln$ (μm). The above data was obtained with the following steps for the variables x and y: $\Delta x = 0.1$ μm; $\Delta y = 1.25 \cdot 10^{-4}$ μm. In addition, $y_w = \ell_w = \infty$.

It follows from Table 1 that linear wear Y and the wear rate dY/dN are nonlinear functions of q, q^0, and λ, the fracture-toughness characteristics K_{fc} and g_0, and the defectiveness of the material $\bar{\mu}$ and $\bar{\sigma}$. It can be concluded that Y is most heavily dependent on q, λ, q^0, $\bar{\sigma}$, and g_0. Calculations show that at $q^0 = 49.03$ and 294.2 MPa, the wear rate $|dY/dN|$ increases with an increase in N. This occurs due to the small difference in the rates of growth of fatigue cracks located at different depths. In other computational variants, $|dY/dN|$ differs little from the values shown in Table 1. It also follows from the results we obtained that a steady rate of wear is not reached within the investigated range of load cycles.

It is interesting to note that the incubation period of the process (see the first and last rows of Table 1) is shorter for those computational variants for which $|dY/dN|$ is greater. The presence of an incubation period is connected with the fact that the probability of the existence of cracks with a half-length $\ell > \ell_k$ during this period is negligibly low (less than 10^{-4} in the calculations). This is a consequence of the rapid increase in the functions $f(0, x, y, \ell_0)$ with an increase in ℓ_0. We conclude from Table 1 that wear intensifies with an increase in q, q^0, λ, $\tilde{\mu}$, $\bar{\sigma}$, g_0 and a decrease in K_{fc}.

It must be noted that, in the cases we examined, we saw a monotonic reduction in the maximum of the stress-intensity factor k_{10} and an increase in the critical half-length $\tilde{\ell}_k$ with an increase in $|y|$ ($y < 0$). Here, for all y the angle of orientation of the cracks α turns out to be very close to $\pi/2$.

TABLE 5

$N\cdot 10^{-24}$	0,461	0,472	0,480	0,487	0,495	0,503	0,536	0,544	0,587	0,595	0,603	0,611
$\|Y\|$, μm	0	0,624	0,673	0,703	0,718	0,725	0,863	0,870	1,023	1,051	1,073	1,080
$N\cdot 10^{-24}$	0,619	0,645	0,653	0,661	0,669	0,678	0,704	0,712	0,720	0,728	0,762	0,770
$\|Y\|$, μm	1,088	1,247	1,275	1,290	1,298	1,305	1,465	1,500	1,515	1,523	1,689	1,718
$N\cdot 10^{-24}$	0,778	0,786	0,821	0,829	0,837	0,845	0,880	0,888	0,896	0,904	0,912	0,938
$\|Y\|$, μm	1,733	1,740	1,907	1,935	1,950	1,958	2,125	2,160	2,175	2,182	2,190	2,349
$N\cdot 10^{-24}$	0,946	0,954	0,963	0,997	1,005	1,013	1,022	1,056	1,064	1,073	1,081	1,089
$\|Y\|$, μm	2,385	2,400	2,407	2,574	2,602	2,617	2,625	2,792	2,827	2,842	2,856	2,864

The wear process is increasingly associated with the process of delamination as the compressive residual stresses q^0 increase. The latter process differs from wear in its evolution and the greater size of the products of disintegration of the material. Thus, at $q^0 = -73.55$ MPa and $\lambda = 0.014$ (with the remaining parameters corresponding to the base variant), wear still takes place (Table 2). Here, the behavior of k_{10} and ℓ_k differ from the case described above. With an increase in $|y|$ $(y < 0)$, there is a sharp reduction in k_{10} to the minimum (Table 3). This reduction is followed by an increase to the maximum value and a subsequent monotonic decrease to zero. The behavior of $\tilde{\ell}_k$ as a function of y is determined by the relation $\tilde{\ell}_k \sim k_{10}^{-2}$. The angle α decreases monotonically with an increase in $|y|$. The wear rate dY/dN is close to constant, which can be attributed to the above-described dependence of k_{10} on y. The values of Δx, y_w, and ℓ_w were the same as previously in the calculations, while $\Delta y = 2.5\cdot 10^{-3}$ μm.

At $q^0 = -196.13$; -490.33 MPa ($\lambda = 0.014$, with the remaining parameters corresponding to the base variant), wear and delamination occur simultaneously (Tables 4 and 5). Here, in contrast to the previous cases, $\Delta y = 1.45\cdot 10^{-2}$ μm.

In the computational variants examined here, Y(N) is a discontinuous piecewise-constant function. Tables 4 and 5 show values of N_i between which $Y(N) = Y_{i+1} = \text{const}$ ($N_i \le N < N_{i+1}$, $i = 0, 1, \ldots$) and at which Y(N) undergoes a discontinuity. With an increase in $|y|$ $(y < 0)$, the value of k_{10} initially increases. It then reaches maxima at $y = -0.558$ and -0.624, after which it decreases to zero. Similar behavior is seen from the angle α, which remains close to zero for any y. The behavior of $\tilde{\ell}_k$ follows from (1.11). The depth of the indicated extrema increases slowly with an increase in $|q^0|$ $(q^0 < 0)$ and λ. We should also note that there are periodic accelerations and decelerations of the wear and delamination processes (see Tables 4 and 5).

As before, an increase in λ is accompanied by an increase in the rates of wear and delamination, while these rates decrease with an increase in $|q^0|$ $(q^0 < 0)$. It follows from Tables 1, 2, 4, and 5 that the wear particles encountered at $q^0 > 0$ and relatively small $|q^0|$ $(q^0 < 0)$ are considerably smaller than the analogous particles seen at substantial compressive stresses q^0. In the latter case, the particles are highly elongated (this follows from the fact that $\alpha \approx 0$ and the critical half-lengths of the cracks are very large). Also, at $q^0 > 0$ and small $|q^0|$ $(q^0 < 0)$, wear is continuous over time. This contrasts with the wear seen at large $|q^0|$ $(q^0 < 0)$, which is many orders greater than wear in the other case just mentioned. The low wear rate is connected with our neglect of various factors (surface roughness, abrasive contaminants in the lubricant, abrasive particles penetrating the surface, etc.) which would tend to intensify the process.

In the calculations performed above, we assumed that the residual stresses $q^0(y)$ were constant in the depth direction. Numerical study of the problem showed that fracture does not begin at $q^0(y) = \text{const}$. Fracture can begin when residual tensile stresses are created at depths (relative to the surface of the body) that are comparable to the size of the contact region. For this to occur, the stress-intensity factors k_1 in the layers undergoing fracture must be at least of the same order of magnitude as in the surface layers subjected to wear.

TABLE 6

$P(N)$	Base variant	$q=1,96$ GPa	$q=0,98$ GPa	$\lambda=0,04$	$\tilde{\mu}=1$	$\tilde{\mu}=2,69$	$\bar{\sigma}=0,2$	$\bar{\sigma}=1$	$g_0=1,87\cdot10^{-11}$ MPa$^{-2m}\cdot$m^{1-m}	$g_0=1,02\cdot10^{-10}$ MPa$^{-2m}\cdot$m^{1-m}
	$N\cdot10^{-10}$									
0,95	0,65	1,75	6,90	10,66	0,91	0,33	0,79	0,46	1,13	0,20
0,90	0,70	1,90	7,45	11,50	0,98	0,36	0,82	0,53	1,22	0,22
0,75	0,79	2,16	8,45	13,05	1,11	0,41	0,86	0,69	1,38	0,25
0,50	0,91	2,48	9,73	15,02	1,28	0,47	0,91	0,91	1,59	0,29
0,25	1,05	2,86	11,21	17,29	1,47	0,54	0,96	1,21	1,83	0,33
0,1	1,19	3,24	12,89	19,63	1,68	0,62	1,01	1,55	2,08	0,38

TABLE 7

$P(N)$	Base variant	$\max\limits_y q^0(y)=19,61$ MPa	$\max\limits_y q^0(y)=49,03$ MPa	$P(N)$	Base variant	$\max\limits_y q^0(y)=19,61$ MPa	$\max\limits_y q^0(y)=49,03$ MPa
	$N\cdot10^{-10}$				$N\cdot10^{-10}$		
0,95	0,65	0,0860	0,0105	0,50	0,91	0,1213	0,0148
0,90	0,70	0,0928	0,0113	0,25	1,05	0,1397	0,0171
0,75	0,79	0,1054	0,0129	0,1	1,19	0,1586	0,0194

In a uniform isotropic elastic half-plane with a stress-free surface, it is impossible to create depthwise-varying residual stresses $q^0(y)$ (except for stresses which change linearly with depth). In practice, various processing factors cause the materials of parts to be nonuniform. This in turn creates residual stresses $q^0(y)$ in the part which vary through its depth. Thus, if we analyze the fracture process without resort to the solutions of the corresponding contact problems of the theory of elasticity for cracked nonuniform bodies,* we approximately assume that $q^0 = q^0(y) \neq \text{const}$ and we employ the dominant terms of the solutions of contact problems for cracked bodies [2] corresponding to uniform and isotropic elastic materials. The accuracy required for practical applications can generally be attained using such assumptions.

Let us examine the fracture process using the example of steel ShKh15, with the above-indicated base-variant parameters. We changed the values of λ and q^0 when we used the base variant to calculate wear: $\lambda = 0.08$ and $q^0(y) = (\alpha_1 + \alpha_2 y)\cdot\exp(\alpha_3 y)$, $\alpha_1 = -490$ MPa, $\alpha_2 = -16.64$ TPa·m^{-1}, $\alpha_3 = 9.5\cdot10^4$ m^{-1}. Here, $q^0(0) = \alpha_1$, $\max q^0(y) = 49.03$ MPa is attained at $y_{max} = -40$ µm. We took $\Delta x = 0.1$ µm, $\Delta y = y_w = 1$ µm, and $\ell_w = \infty$ in the calculations. Table 6 – structured in the same way as Table 1 except for the replacement of $|Y(N)|$ by $P(N)$ – shows the results of several variants of calculation of the probability of the absence of fracture $P(N)$. As $|Y(N)|$, $P(N)$ is a nonlinear function of q, λ, $\max q^0(y)$, y_{max}, K_{fc}, g_0, $\tilde{\mu}$, and $\bar{\sigma}$. The data shown in the table indicates that $P(N)$ is most heavily dependent on λ and q. The calculations also showed that $P(N)$ depends the least on y_{max}. The fracture process intensifies with an increase in q, $\max q^0(y) > 0$, λ, $\tilde{\mu}$, $\bar{\sigma}$, g_0 and a decrease in $|y_{max}|$ ($y_{max} < 0$), K_{fc}. Wear $Y(N)$ is absent for the values of N indicated in Table 6. Here, the stress-intensity factor k_{10} increases with an increase in $|y|$ ($y < 0$) and reaches a maximum at a depth of 3-7 µm. It then declines to a minimum located near the ordinate $y_0 = -\alpha_1/\alpha_2$ where $q^0(y)$ vanishes. This ordinate is generally located in the region of compressive residual stresses. With a further increase in $|y|$ ($y < 0$), the coefficient $k_1(y)$ increases to a maximum located on the interval $[y_0, y_{max}]$ and then monotonically decreases. In the cases shown in Table 6, the value of $\max\limits_y k_{10}(y)$ at the deeper maximum is more than two orders greater than the value at the maximum near the surface. The calculations showed that fracture is governed by the deeper maximum, which is attributable to the fact that cracks grow considerably more rapidly at this depth than at the other points.

Cracks grow nearly parallel to the surface of the body in the layer of material $y_0 \le y < 0$, while at $y < y_0$ their angle of orientation α becomes close to $\pi/2$.

*If such solutions are available for the given problems, they can be successfully incorporated into the proposed model.

At fixed $q^0(0)$ and y_{max}, an increase in $\max_y q^0(y)$ leads to a sharp increase in the rate of fracture. This is illustrated by the data in Fig. 7, where the first column shows the number of cycles N at which the specified probability P(N) is attained for the base variant. The second and third columns show the same for the base variant with $\max_y q^0(y) = 19.61$ MPa and $\max_y q^0(y) = 49.03$ MPa, respectively. It should be noted that with a further increase in $\max_y q^0(y)$, the probability P(N) is determined with a high degree of accuracy only by the indicated maximum.

The results described above qualitatively – and in some cases quantitatively – agree with experimental data. A direct comparison is generally difficult because of the paucity of literature data showing the results of contact fatigue tests and the corresponding initial characteristics of the model examined here.

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BUCKLING OF A NONLINEARLY ELASTIC SLAB LYING ON THE SURFACE OF A LIQUID WITH ALLOWANCE FOR PHASE TRANSFORMATION

V. A. Eremeev

UDC 539.3

The equations of the three-dimensional nonlinear theory of elasticity [1] are used to formulate equilibrium conditions with finite strains for an arbitrary thermoelastic body undergoing a phase transformation. These conditions are then used to study the equilibrium of a circular uniform slab lying on the surface of a melt in a gravitational field. We use the model of a non-Hookian material as the governing relation for the material of the slab, this model being one possible generalization of the model of an incompressible linearly elastic body to the case of finite strains. The method of superimposing a small strain on a finite strain [1] is used to study local loss of stability of the slab due to its compression in the radial direction. The critical strains are determined numerically. A similar approach is used to study buckling of the slab in the absence of phase transformation.

1. We will examine the equilibrium of a thermoelastic body undergoing a first-order liquid—solid phase transformation. Similar transitions were studied within the framework of continuum mechanics in [2-7], where various approaches were employed to obtain relations describing the phase transition at the phase boundary. A characteristic feature of the problem of the equilibrium of a thermoelastic body under phase-transformation conditions is the presence of an a priori unknown phase boundary. As an auxiliary phase-transformation condition serving to determine the position of the phase boundary, we choose the equation of the fusion curve [8]. This equation expresses the dependence of the melting point on pressure in the liquid [8].

Let the volume occupied by the body in the reference configuration be equal to v. We represent the external boundary of the body as the union of the surface γ separating the body from the liquid and the surface $\sigma = \sigma_1 \cup \sigma_2 \cup \sigma_3 = \sigma_4 \cup \sigma_5$ (Fig. 1). The body is de-

Rostov-on-Don. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 141-147, May-June, 1991. Original article submitted July 12, 1989; revision submitted January 22, 1990.